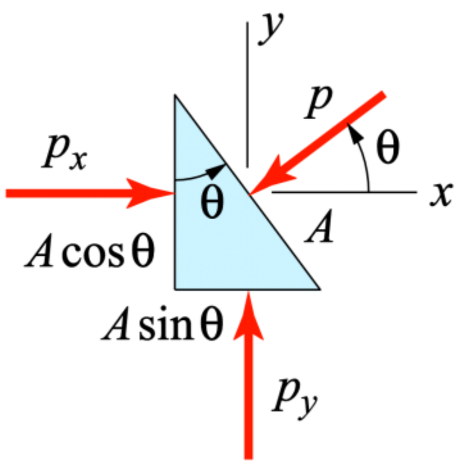
Fluids

Pascal's law: A fluid at rest creates a pressure p at a point that is the ***same*** in ***all*** directions

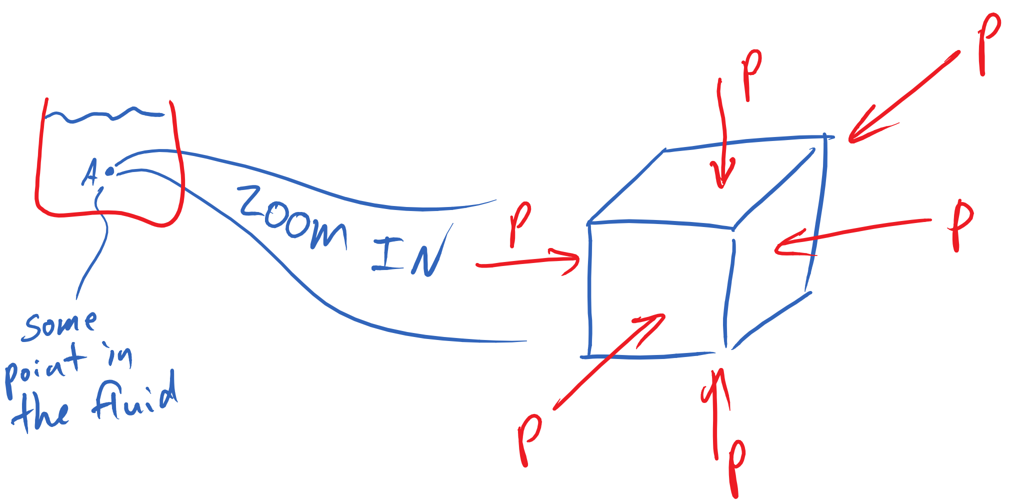


For equilibrium of an infinitesimal element,

$$\Sigma F_x = 0: \quad p_x(A\cos\theta) - pA\cos\theta = 0 \quad \Rightarrow \quad p_x = p,$$
$$\Sigma F_y = 0: \quad p_y(A\sin\theta) - pA\sin\theta = 0 \quad \Rightarrow \quad p_y = p.$$

Thus, $p_x = p_y = p$ for any angle θ. The Pascal's law holds for fluids, not solids.

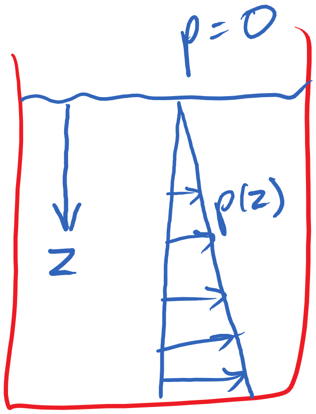
Incompressible: An incompressible fluid is one for which the mass density ρ is independent of the pressure p. Liquids are generally considered incompressible. Gases are compressible, but may be approximated as incompressible if the pressure variations are relatively small.



Fluid is at rest!

p acts ⊥ to all surfaces!

In a liquid → incompressible fluid
⇒ density ρ is constant
with a free surface exposed to the air



$p = 0$ at the surface
and p increases linearly with depth z.

$$p(z) = 0 + \rho \cdot g \cdot z$$

$$\boxed{p(z) = \rho \cdot g \cdot z}$$

ρ = mass density $\left(\frac{\text{mass}}{\text{volume}}, \frac{\text{mass}}{L^3}\right)$

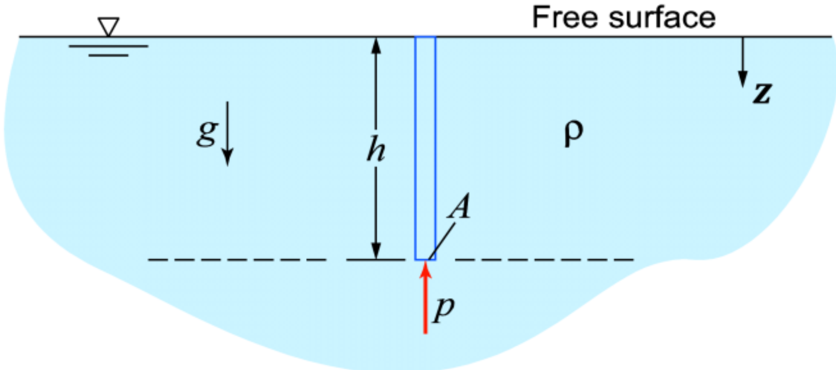
g = gravitational accel. $\left(\frac{L}{\text{time}^2}\right)$

Dimensions of ρg: $\frac{\text{force}}{\text{volume}}$

Dim's of $\rho g z$: $\frac{\text{force}}{\text{area}}$ pressure!

Fluid Pressure

For an incompressible fluid at rest with mass density ρ, the pressure varies linearly with depth z



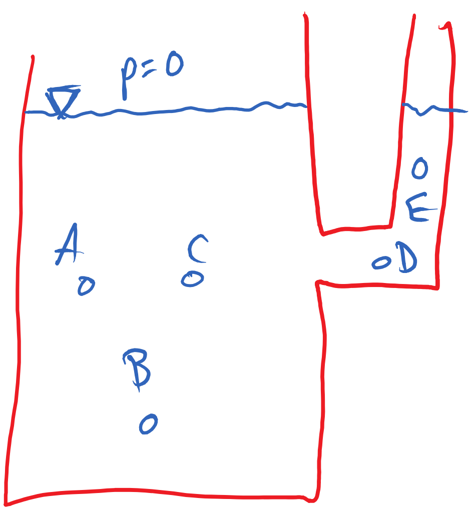
Summing forces in the vertical direction gives

$$\Sigma F_x = 0: \quad mg - pA = 0 \quad \Rightarrow \quad (\rho(Ah))g - pA = 0 \quad \text{or} \quad p = \rho g h\,.$$

In general, this result is written as $\boxed{p = \rho\, g\, z = \gamma\, z}$

where $\gamma = \rho\, g$ is called the specific weight (weight per unit volume).

For fresh water: γ =62.4 lb/ft^3 (9810 N/m^3)



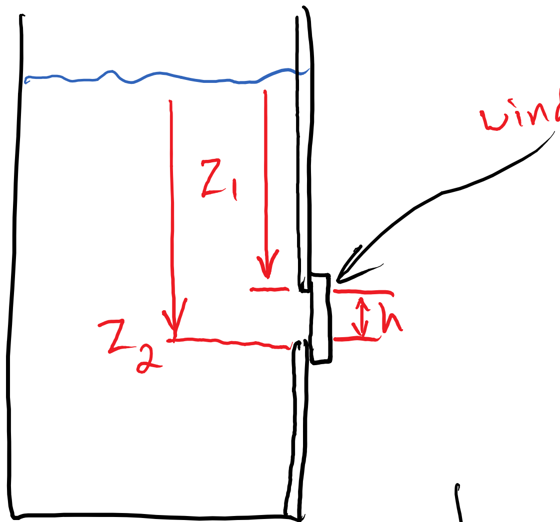
p is greatest: B

p is lowest: E

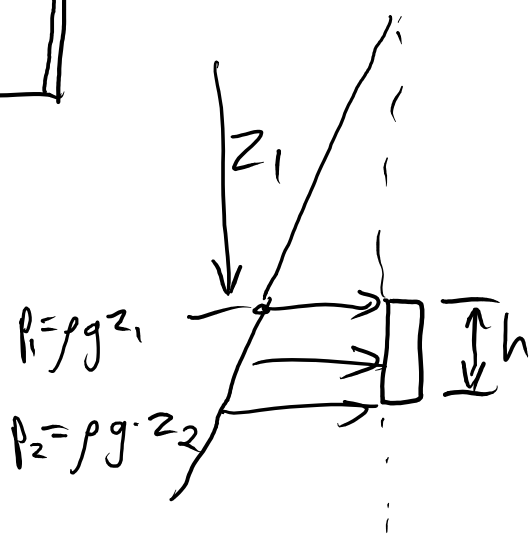
p varies in z, not x & y.

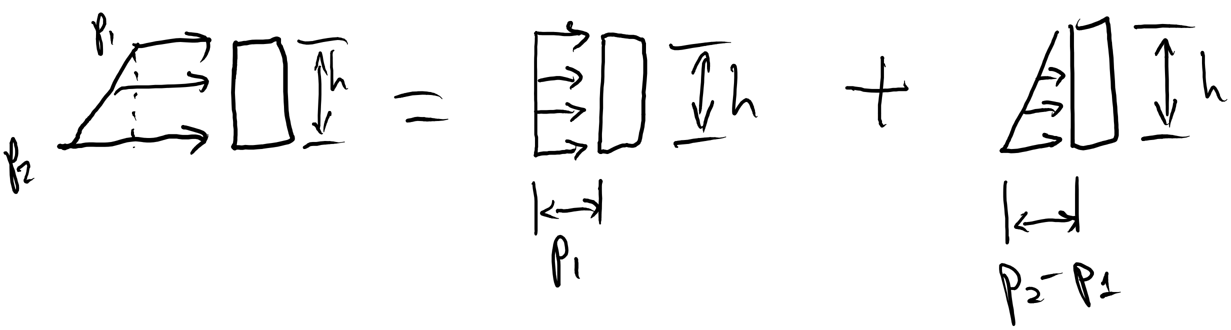
Force against a surface

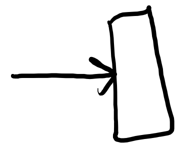
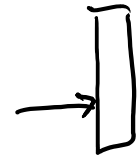
$$F = \int p(z) \cdot dA$$



What force acts on the window? (suppose the window has a width w)





$= F_{\square}$  $+ F_{\Delta}$ 

$$F_{\square} = p_1 \cdot \overbrace{(h \cdot w)}^{A}$$

$$F_{\Delta} = \frac{1}{2}(p_2 - p_1) \cdot \overbrace{h \cdot w}^{A}$$

$$p_1 = \rho g z_1$$

$$p_2 = \rho g z_2$$

$$p_2 - p_1 = \rho \cdot g \cdot (z_2 - z_1)$$

$$= \rho \cdot g \cdot h$$

$$F_{\Delta} = \frac{1}{2} \cdot \rho \cdot g \cdot h^2 \cdot w$$

$$F_p = \sum F = F_{\square} + F_{\Delta}$$

$$= \rho \cdot g \cdot z_1 \cdot h \cdot w + \frac{1}{2} \rho \cdot g \cdot h \cdot (h \cdot w)$$

$$\boxed{F_R = \rho \cdot g \cdot h \cdot w \cdot \left(\frac{h}{2} + z_1\right)}$$

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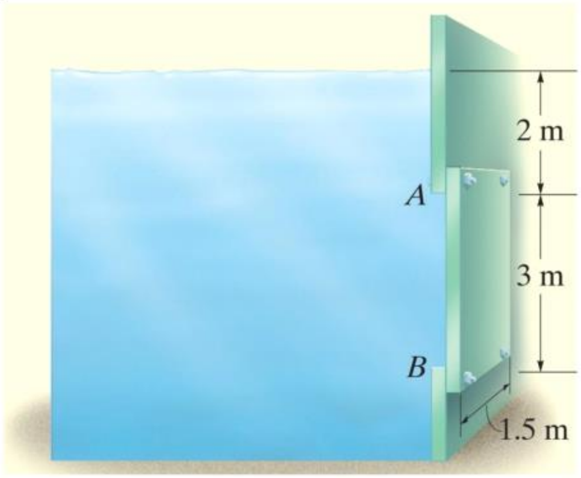


Determine the magnitude and location of the resultant hydrostatic force acting on the submerged rectangular plate AB. The plate has width 1.5m. The density of the water is 1000 kg/m³

$\rho = 1000\ kg/m^3$

$h = 3m$

$z_1 = 2m$

$w = 1.5m$

Magnitude

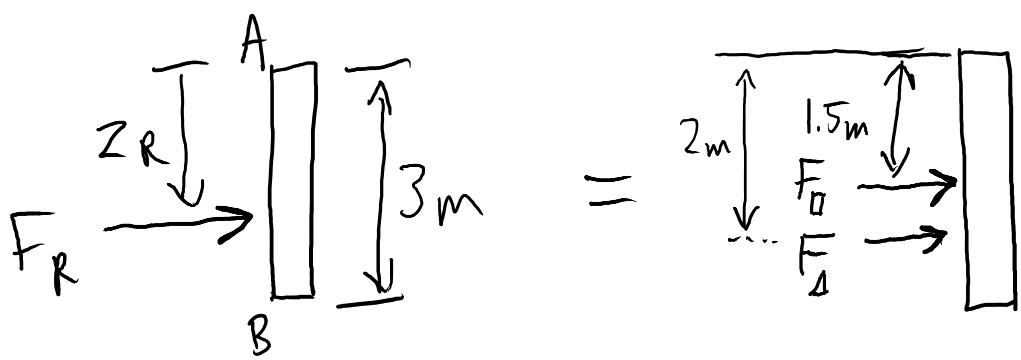
$$F_R = \rho \cdot g \cdot h \cdot w \cdot \left(\frac{h}{2} + z_1\right)$$

$$\boxed{F_R} = \left(1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right)(3m)(1.5m)\left(\frac{3m}{2} + 2m\right)$$

$$= \left(9810\frac{N}{m^3}\right)\left(4.5m^2\right)(3.5m)$$

$$= 154507.5N = \boxed{155kN}$$

Location of resultant



$$F_R \cdot x_R = 1.5m \cdot F_\square + 2m \cdot F_\Delta$$

$$z_R = \frac{(1.5m) \cdot F_{\square} + (2m) \cdot F_{\triangle}}{F_R}$$

$$F_{\square} = \rho \cdot g \cdot z_1 \cdot h \cdot w$$

$$= \left(1000 \tfrac{kg}{m^3}\right)\left(9.81 \tfrac{m}{s^2}\right)(2m)(3m)(1.5m)$$

$$= \left(9810 \, N/m^3\right)\left(9 \, m^3\right) = 88290 \, N$$

$$F_{\triangle} = \frac{1}{2} \cdot \rho \cdot g \cdot h^2 \cdot w$$

$$= \frac{1}{2}\left(1000 \, kg/m^3\right)\left(9.81 \tfrac{m}{s^2}\right)(3m)^2(1.5m)$$

$$= 66217.5 N$$

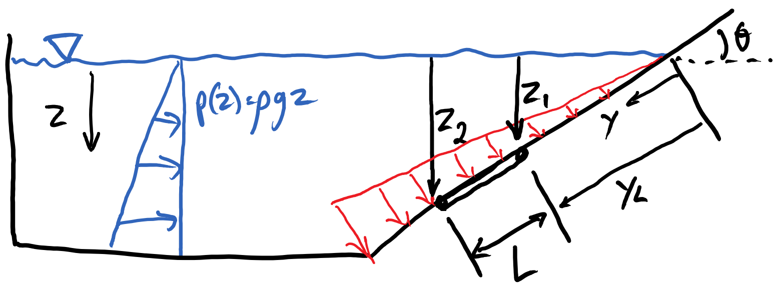
$$z_R = \frac{(1.5m) \cdot F_{\square} + (2m) \cdot F_{\triangle}}{F_R}$$

$$= \frac{(1.5m) \, 88290 + (2m) \cdot 66217.5}{88290 + 66217.5}$$

$z_R = 1.714 \, m$ below point A

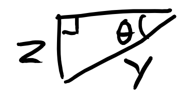
$$\boxed{\begin{array}{l} F_R = 155\,\text{kN} \\ \text{acts } 3.71\,\text{m below the} \\ \text{free surface} \end{array}}$$

Force on inclined surface



pool has a width w

$$p = p(z) = \rho \cdot g \cdot z$$



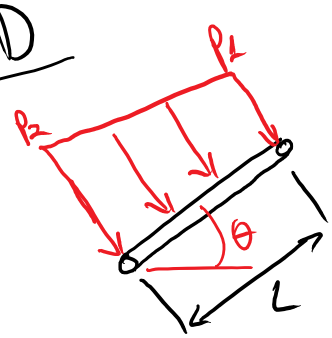
$$z = y \cdot \sin\theta$$

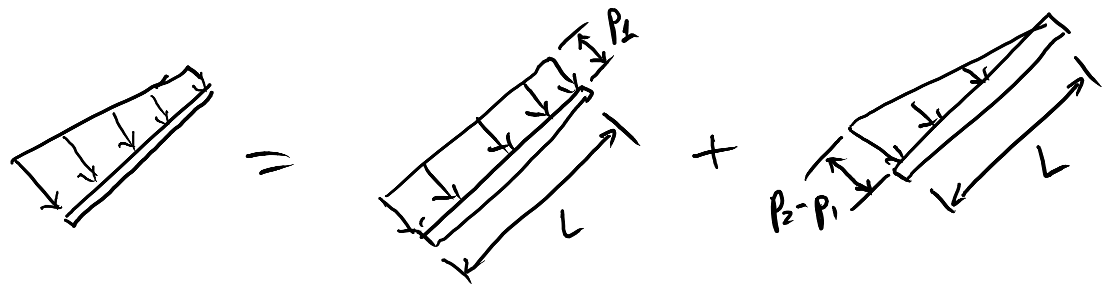
$$z_1 = y_1 \cdot \sin\theta$$

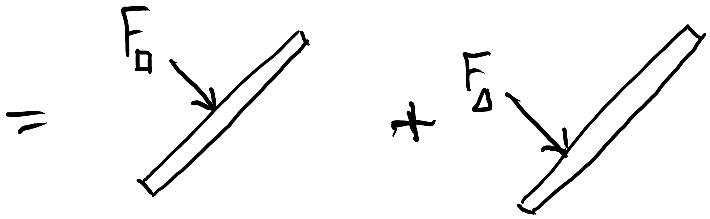
$$z_2 = y_2 \cdot \sin\theta$$

$$L = y_2 - y_1$$

FBD







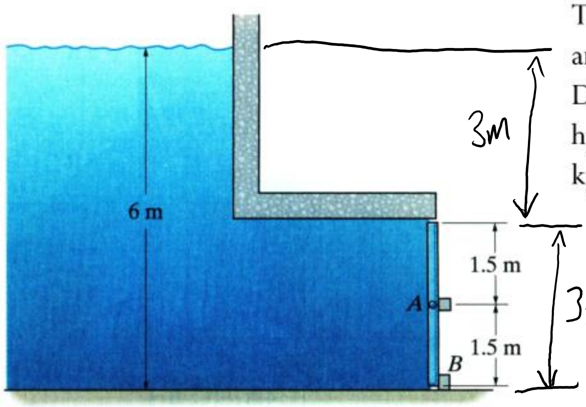
$$F_R = \Sigma F = F_{\square} + F_{\Delta}$$

$$F_{\square} = p_1 \cdot L \cdot w = \rho g z_1 \cdot L \cdot w = \rho g \cdot \underline{y_1} \cdot \sin\theta \cdot L \cdot w$$

$$F_{\Delta} = \frac{1}{2}(p_2 - p_1) \cdot L \cdot w = \frac{1}{2} \cdot \rho \cdot g \cdot (y_2 - y_1) \cdot \sin\theta \cdot L \cdot w$$

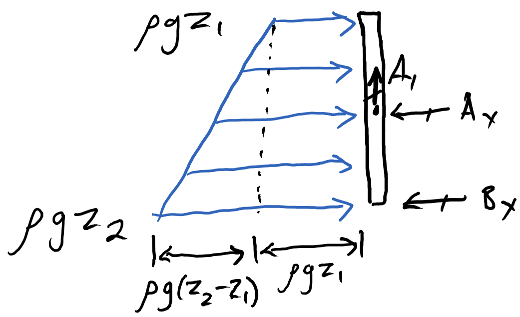
$$= \frac{1}{2} \rho \cdot g \cdot \sin\theta \cdot L^2 \cdot w$$

$$\boxed{F_R = \rho \cdot g \cdot \sin\theta \cdot L \cdot w \cdot \left(y_1 + \frac{1}{2} L\right)}$$



The 2m wide rectangular gate is pinned at its center A and is prevented from rotating by the block at B. Determine the reactions at these supports due to hydrostatic pressure. The density of the water is 1000 kg/m³

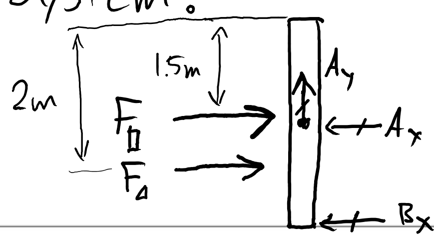
FBD of gate



$z_1 = 3m$

$z_2 = 6m$

Equivalent System:



$(\Sigma M)_A = 0$

$\Rightarrow (0.5m)F_\triangle - (1.5m)B_x = 0$

$\Rightarrow B_x = \frac{0.5}{1.5} \cdot F_\triangle = \frac{F_\triangle}{3}$

$F_\triangle = \frac{1}{2} \cdot \rho \cdot g \cdot \underbrace{(z_2 - z_1)}_{} \cdot \overbrace{(3m)}^{z_2 - z_1} \overbrace{(2m)}^{w}$

$= \frac{1}{2}(1000 \, kg/m^3)(9.81 \, \frac{m}{s^2})(3m)^2(2m)$

$= 88290 \, N$

$\boxed{B_x = \frac{F_\triangle}{3} = 29430 \, N}$

$F_\square = \rho \cdot g \cdot z_1 \cdot (z_2 - z_1) \cdot w$

$= (1000 \, kg/m^3)(9.81 \, m/s^2)(3m)(3m)(2m)$

$= 176{,}580 \, N$

$\Sigma F_x = 0$

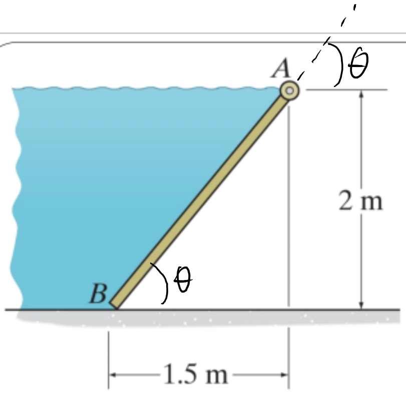
$$\Rightarrow \quad F_{\square} + F_{\Delta} - A_x - B_x = 0$$

$$A_x = F_{\square} + F_{\Delta} - B_x$$

$$= (176580\,N) + (88290\,N) - (29430\,N)$$

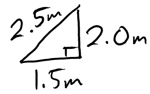
$$= 235440\,N$$

$$\boxed{A_x = 235\,kN}$$



Determine the magnitude of the resultant hydrostatic force acting on the gate AB. The gate has width 1.5m. The density of the water is 1 Mg/m^3

$$\sin\theta = \frac{2.0}{2.5} = \frac{4}{5}$$



From previous example:

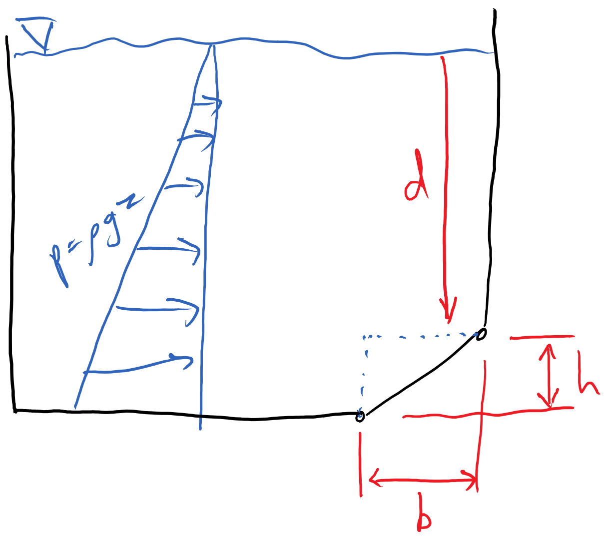
$$F_R = \rho \cdot g \cdot \sin\theta \cdot L \cdot w \cdot \left(y_1 + \tfrac{1}{2}L\right)$$

set $y_1 = 0$, $L = 2.5\,m$, $w = 1.5\,m$

$$F_R = \left(1\,\tfrac{Mg}{m^3}\right)\left(9.81\,\tfrac{m}{s^2}\right)\left(\tfrac{4}{5}\right)(2.5m)(1.5m)\left(0 + \tfrac{2.5m}{2}\right)$$

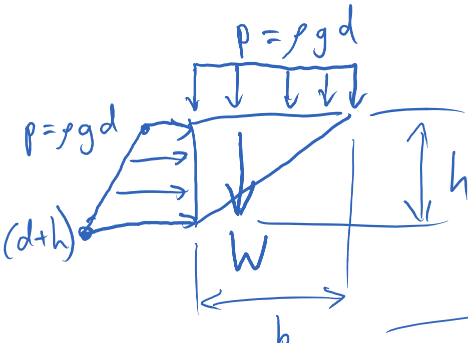
$$= \left(9810\,\tfrac{N}{m^3}\right)\left(\tfrac{4}{5}\right)\left(4.6875\,m^3\right)$$

$$= \boxed{36.8\,kN}$$



Find the resultant force on the angled corner

Take FBD of the water prism



$$\Sigma F_x = \underbrace{\rho g \cdot d \cdot h \cdot w} + \tfrac{1}{2}(\rho \cdot g \cdot h) \cdot h \cdot w = \boxed{\rho g h w \left(d + \tfrac{h}{2}\right)}$$

$$\Sigma F_y = -W - \overbrace{\rho \cdot g \cdot d \cdot (b \cdot w)}^{\text{rectangular portion of the pressure}}$$

$$W = \rho \cdot g \cdot \text{Volume} = \rho \cdot g \cdot \Big(\overbrace{\tfrac{1}{2} \cdot b \cdot h}^{\text{area of triangle}} \cdot \overset{\text{width of pool}}{w} \Big)$$

$$= -\rho \cdot g \cdot b w \cdot \left(\frac{h}{2} + d \right)$$

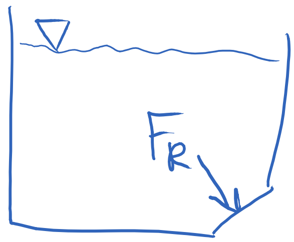
Magnitude of resultant force on the inclined surface

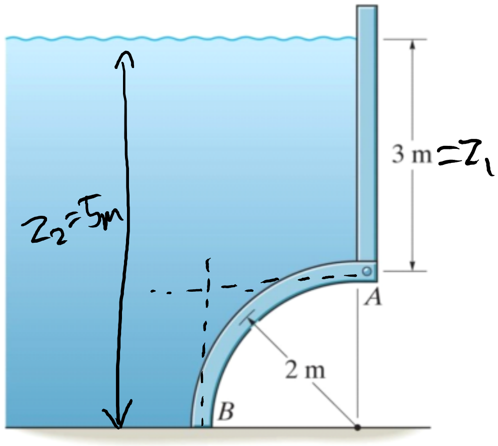
$$|F_R| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{\left[\rho g h w \left(d + \frac{h}{2}\right)\right]^2 + \left[-\rho g b w \cdot \left(\frac{h}{2} + d\right)\right]^2}$$

$$= \rho \cdot g \cdot w \sqrt{h^2 \left(d + \frac{h}{2}\right)^2 + b^2 \left(d + \frac{h}{2}\right)^2}$$

$$\boxed{|F_R| = \rho \cdot g \cdot w \cdot \left(d + \frac{h}{2}\right) \sqrt{h^2 + b^2}}$$

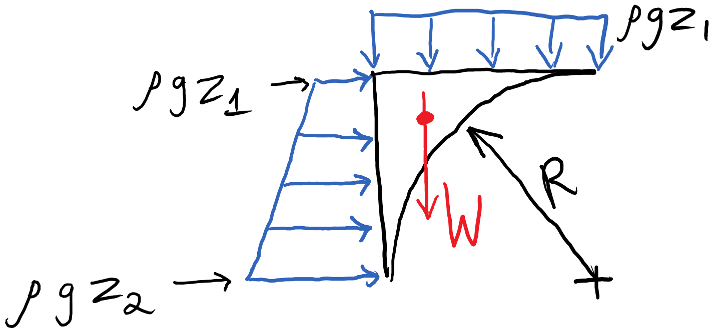

F_R



The arched surface AB is shaped in the form of a quarter circle. If it is 8 m long, determine the horizontal and vertical components of the resultant force caused by the water acting on the surface. The density of the water is 1 Mg/m^3

FBD of superscribing square about quarter-circle

$w = 8m$



$$W = \rho g w \left(R^2 - \frac{\pi R^2}{4}\right) = \rho \cdot g \cdot w \cdot \left(\frac{4-\pi}{4}\right) R^2$$

$$W = \left(1 \tfrac{Mg}{m^3}\right)\left(9.81 \tfrac{m}{s^2}\right)(8m)\left(\frac{4-\pi}{4}\right)(2m)^2$$

$$W = 67{,}367 \text{ N}$$

$$\boxed{\Sigma F_y = -\rho \cdot g \cdot z_1 \cdot R \cdot w - W}$$

$$= -\left(9810 \tfrac{N}{m^3}\right)(3m)(2m)(8m) - 67367 \text{ N}$$

$= -470.9\,kN - 67.4\,kN$

$= \boxed{-538.3\,kN}$

$$\boxed{\Sigma F_x} = \rho \cdot g \cdot R \cdot w \cdot \left(z_1 + \frac{R}{2}\right) = \left(9810\,\frac{N}{m^3}\right)(2m)(8m)\left(3m + \frac{2m}{2}\right)$$

$$= \left(9810\,\frac{N}{m^3}\right)\left(64\,m^3\right)$$

$$= 627840\,N$$

$$= \boxed{627.8\,kN}$$